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Simulation of the vortex motion in high-T_c superconductors

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Abstract. 1D and 2D simulations of the single-vortex dynamics in the presence of a random pinning potential and a periodic potential have been carried out. It is shown that the randomness of the pinning site distribution does not have much effect on the transport properties such as the I–V characteristics of the high-T_c superconductors, which has been widely discussed in the approximation of a periodic pinning potential using an analytical method. The randomness effect probably reduces greatly the vortex diffusing mobility only below the depinning current value; this is more obvious at low temperatures.

There have been many efforts dedicated to vortex dynamics in high-T_c superconductors in the last few years because of its importance in trying to find materials with a higher critical current density. Vortex dynamics in the superconductors tells us information about the motion of the vortices influenced by various interactions, including important pinning effects. Usually, the pinning is caused by inhomogeneities present in the superconductors, e.g. impurities and defects. However, in addition to the traditional pinning centres, oxide superconductors with a characteristic layered structure have their own intrinsic pinning when the vortices move in a direction perpendicular to the layers [1]. In conventional superconductors, vortex dynamics is usually studied in two typical cases: flux creep and flux flow. The successful classical Anderson–Kim [2] thermally activated flux creep model and the Bardeen–Stephen [3] model have been used to describe them, respectively. In high-T_c superconductors, the thermal energy $k_B T$ is rather high and usually comparable with the pinning energy; so the simple flux creep description is only applicable in the low-temperature region. When the thermal energy $k_B T$ becomes comparable with the pinning energy, both flux creep and flux flow will dominate the vortex dynamics and finding a correct description of the vortex motion in this regime is difficult. Inui *et al* [4] used a single-flux-depinning model to interpret the resistive broadening in high-T_c superconductors in which they neglected the random distribution of the impurities and for simplicity took approximately a sinusoidal form to represent the position distribution of pinning sites. More recently, by taking the thermally fluctuating force into account explicitly, we have successfully explained the widely observed power-law I–V characteristics over the whole temperature region [5]. In our model a sinusoidal form of pinning potential was also assumed, which is more suitable for intrinsic pinning with the magnetic field parallel to the planes.

More generally, however, the problem is complicated by the fact that, because of inhomogeneities in the materials, the flux line always experiences a random potential background, and the vortex mobility is thus determined by the combined effect of the random pinning potential and thermal fluctuations. Therefore, it is interesting and important to investigate the effect on the flux motion due to the random distribution of pinning sites

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in materials. In this paper, we simulate the vortex diffusion in one and two dimensions in the presence of randomly distributed pinning sites and thermal noise. We find that the main I–V features in the mixed state of superconductor have not changed much because of the presence of randomness in the distribution of pinning sites, which justifies the many approximations used in the previous papers.

The dynamic equation of a single vortex is expressed as the following:

$$\eta\,\mathrm{d}\boldsymbol{r}/\mathrm{d}t = (\boldsymbol{F}_\mathrm{d} + \boldsymbol{F}_\mathrm{p}) + \tilde{\boldsymbol{L}}(t) \tag{1}$$

where η is the viscosity coefficient, $\boldsymbol{F}_\mathrm{d} = (1/c)\boldsymbol{J}\phi_0$ is the driving force with $\boldsymbol{J}$ being the current density and ϕ_0 the superconducting flux quantum, $\boldsymbol{F}_\mathrm{p}$ is the pinning force and $\tilde{\boldsymbol{L}}(t)$ is the fluctuating force which may be due to the random Lorentz force caused by thermal motion of the normal electrons in the vortex core. Here we consider that $\boldsymbol{F}_\mathrm{p}$ is caused by the interaction of the vortex with a number of pinning centres randomly positioned at $\boldsymbol{R}_i$ and as usual we choose a Gaussian form of the individual pinning wells:

$$U_\mathrm{p} = A_\mathrm{p}\sum_i \exp\left(\frac{-(x-x_i)^2}{\xi^2}\right) \tag{2}$$

where the amplitude A_p is the condensation energy stored in the vortex core, i.e. $A_\mathrm{p} = (H_\mathrm{c}^2/8\pi)\xi_{ab}^2\xi_c$. The stochastic force $\tilde{L}(t)$ is assumed to be Gaussian white noise. In a Gauss–Markov process, the times t_i between two random-noise pulses are distributed as $p(t_i) = (1/\tau)\exp(-t_i/\tau)$, where τ is the mean time between two pulses. In a system with a discrete grid of time steps Δ, the probability p that after Δ one random pulse acts on the vortex is given by $p = \int_0^\Delta p(t_i)\,\mathrm{d}t = 1 - \exp(-\Delta/\tau) \simeq \Delta/\tau = p$. We find that $\tilde{L}(t)/\eta$ can be written as

$$\left(\frac{2\Delta k_\mathrm{B}T}{\eta p}\right)^{1/2}\sum_j \delta(t-t_j)\gamma(t_j)\Theta(p-q_j)$$

where j labels the jth time step, $\gamma(t_j)$ is a random number chosen from Gaussian distribution of mean 0 and width 1, and q_j is just a random number uniformly distributed between 0 and 1. $\Theta(x)$ is defined by

$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0. \end{cases}$$

Combining the results described above, the one-dimensional (1D) discretized equation can be written as

$$x^{n+1} = x^n + [(F_\mathrm{d}^n + F_\mathrm{p}^n)/\eta]\Delta + (2\Delta k_\mathrm{B}T/\eta p)^{1/2}\gamma_x\Theta(p-q_j). \tag{3}$$

We used the algorithm described above to simulate the vortex diffusion in a model superconductor for a system having 200 pinning sites and a length of 200 times the coherence length ξ with a periodic boundary condition. The parameters in our calculation are chosen as follows: $\xi_{ab}(0) = 27$, $\xi_c(0) = 12$, $\rho_n(T_\mathrm{c}) = 2\times10^{-4}\ \Omega\ \mathrm{cm}^{-1}$, $H_{c2}(0) = 127$ T, $H_\mathrm{c}(0) = 2.72$ T where η is determined from the Bardeen–Stephen formula and it is easy to obtain

$$\eta = (h\xi_\mathrm{c}/2c\rho_\mathrm{n})[H_{c2}(0)/H][(1-t)^{1/2}/t]$$

where ρ_n is the normal-state resistivity and we postulate that $\xi_{ab,c}(t) = \xi_{ab,c}(0)(1-t^2)^{-1/2}$, $H_c(t) = H_c(0)(1-t^2)$, $H_{c2}(t) = H_{c2}(0)(1-t)$; here $t = T/T_c$ is a reduced temperature. Now, we should also choose Δ and τ. To do so, we know that the first and second moment can be obtained from equation (1):

$$M_1 \equiv (1/\tau)\langle\delta x\rangle = (1/\eta)F \qquad M_2 \equiv (1/\tau)\langle(\delta x)^2\rangle = 2k_B T/\eta. \tag{4}$$

Then, following the algorithm developed by Brass and Jensen [6], Δ and τ can be selected by calculating the same moments and letting them agree with those given in equation (3). They satisfy the following conditions:

$$\Delta \ll 2k_B T\eta/\langle F\rangle^2 \ll \tau$$

where $\langle F\rangle$ is the average net deterministic force on a flux.

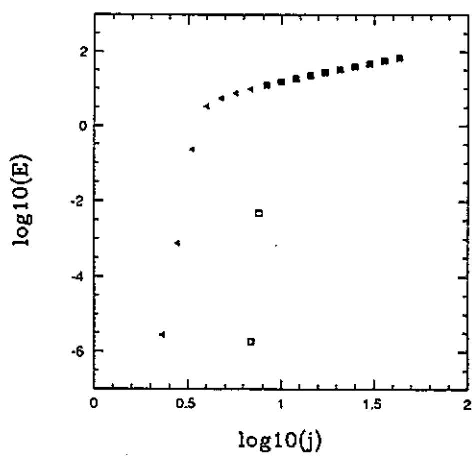

Figure 1. Calculated curves of electric field versus current density in the 1D case for both the random (□) and the periodic (▲) pinning situation at $t = 0.76$.

We obtain the J–E characteristics in figure 1 for both a random and a periodic pinning potential at a temperature $t = 0.76$. The same features are shown in figure 2 for $t = 0.92$. Here $E = (B/C)\langle(\mathrm{d}x/\mathrm{d}t)\rangle$ represents the induction electric field. From these figures, we can note the following points.

(1) A critical current density J_{cr} exists below which the flux mobility almost suddenly falls to zero for both cases, irrespective of whether the pinning potential is random or periodic. Of course, the critical value of the current density for random pinning is higher than that for the periodic situation, which is more obvious at low temperatures, as seen in figure 1. This is because, as shown in figure 3, the amplitude of the random pinning potential has many peaks higher than the amplitude of the periodic potential, and they prevent the vortex from diffusing. This phenomenon may have some connection with the so-called glass state [7] in which all the vortices can be pinned. If the interaction between the vortices is included, which is not considered in this short paper, perhaps the true glass

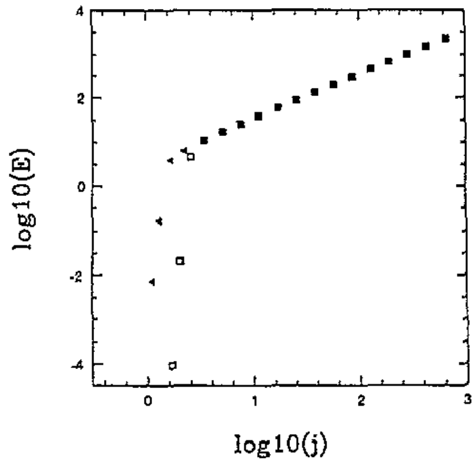

Figure 2. Same as figure 1, but for $t = 0.92$.

state will be able to appear. This interesting problem will be discussed in a forthcoming paper.

(2) Above J_{cr}, the two curves completely coincide with each other. This means that the random distribution of the pinning sites in a superconductor has no significant effect on the flux motion and so the approximation used in many analytical works, i.e. neglecting the randomness in the pinning site distribution and simply choosing a sinusoidal pinning potential, is reasonable, especially at high temperatures.

(3) The power-law J–E characteristics seen in these figures for different temperatures are consistent with the analytical work [8] and experiments [9]. This probably demonstrates that the dependence of the pinning potential on current is logarithmic in a rather wide temperature region [10]. However, we prefer to think that it is caused by the dynamical equation incorporating both flux creep and flux flow naturally, as shown in our analytical work [5] and by the numerical simulation in this paper. This is because we never include the dependence of the pinning potential (random or periodic) on the current density J. We think, most probably, that this power-law behaviour has some deeper intrinsic relation with the so-called self-organized criticality [11, 12], shown in a Langevin equation followed by the vortex moving in a periodic potential or more generally in a dynamical equation with a random potential.

In addition to the 1D simulation, we have also made a similar calculation for the two-dimensional (2D) case. From equation (1), it is easy to obtain a discretized equation for the 2D situation. The equation is

$$\begin{aligned} x^{n+1} &= x^n + [(F_{dx}^n + F_{px}^n)/\eta]\Delta + (2\Delta k_B T/\eta p)^{1/2}\gamma_x \Theta(p - q_j) \\ y^{n+1} &= y^n + (F_{py}^n/\eta)\Delta + (2\Delta k_B T/\eta p)^{1/2}\gamma_y \Theta(p - q_j). \end{aligned} \tag{5}$$

In this paper, we include only the Lorentz force as the driving force $\boldsymbol{F}_d$, and consider that the current flows along the y direction. Therefore, in equation (5) the y component F_{dy}^n of the driving force should be zero. The calculation result is shown in figure 4 for two different temperatures, from which no significant differences from the 1D case are found. The J–E behaviour is very similar to that in the 1D case. However, in this work, we do not include the Magnus force in our equation of motion. So, the driving force in the y direction is very weak and is given by only the thermal noise. Therefore, including the Magnus force will make the motion of flux become of a more 2D nature. Whether or not the situation becomes different after the Magnus force is taken into account is not known at present and will be left for future study.

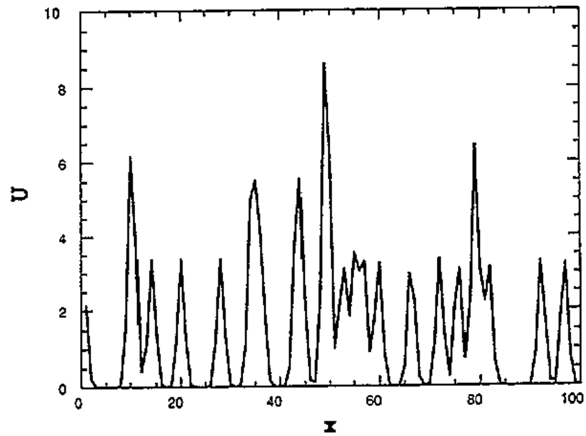

Figure 3. Random pinning potential U versus position x in one dimension.

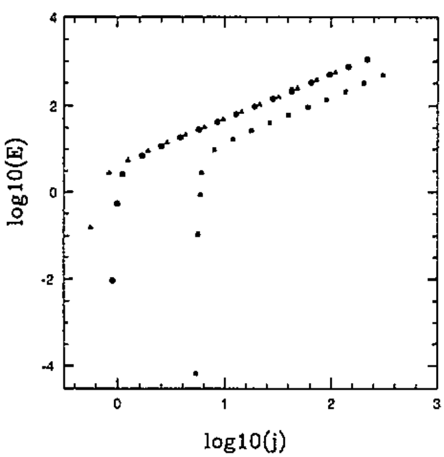

Figure 4. Calculated curves of electric field versus current density in the 2D case for random (●) and periodic (▲) pinning potentials at $t = 0.96$ and for only random pinning (□) at $t = 0.76$.

In conclusion, we have simulated the vortex diffusion in the presence of random pinning potentials for 1D and 2D cases and in particular have discussed the J–E characteristics. We find that the results obtained are similar to those found before using the periodic potentials. Finally, we would like to emphasize that our simulation is in the context of single-vortex dynamics and we include only the combined effects from randomness in the pinning sites distribution and thermal fluctuation. In some cases, single-vortex dynamics will not be enough; the collective effect will be important. The effects of the collective flux motion are also, in our opinion, very interesting and important. In this case, will the superconducting glass state really exist? Is there any 'truly' superconducting state? These questions remain open for future work to resolve.

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